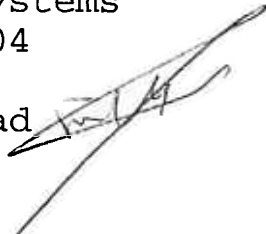


AMERICAN UNIVERSITY OF BEIRUT  
 FACULTY OF ENGINEERING AND ARCHITECTURE  
 EECE 460 Control Systems  
 Spring 2003-2004  
 Quiz II  
 Prof. Fouad Mrad



**Name :**

1.5 hours.

May 21, 2004

Total of 100 points

Open Book Exam, 2 pages

**YOU MUST RETURN THIS EXAM WITH YOUR ANSWER BOOKLET**

Problem 1 (30 points):

The space station attitude control dynamics has the plant transfer function given by  $G(s)$ . Design a digital controller to have desired closed loop natural frequency around 0.3 rad/sec and damping ratio of 0.7 using emulation.

Assume that the supplied Continuous controller is  $D(s)$  and the sampling frequency is 6 rad/sec.

$$G(s) = \frac{1}{s} \quad \text{and} \quad D(s) = \frac{0.81(s+0.2)}{s+0.2}$$

- +3 1. Is the sampling frequency fast enough for emulation? Justify. *OK  $T_s = 1 \text{ sec}$   $\omega_s \approx 20 \text{ rad/sec}$*  (4.6/21.8)
- +8 2. What are the desired transient specifications of the system?  *$\zeta = 0.7$   $\omega_n = 0.3$*
- +8 3. What are the corresponding s-plane desired poles?  *$s_{1,2} = -0.175 \pm j0.24$*
- +8 4. Design  $D^*(z)$  using Zero Pole Matching discretization.  *$0.329 \frac{z-0.82}{z-0.125}$*
- +8 5. Derive the equivalent algorithm (difference equation) for microprocessor coding.  *$u(k) = 0.175 u(k-1) + 0.329 e(k) - 0.719 e(k-1)$*
- +3 6. Is your derived routine implementable? Justify, if not suggest a practical solution. *yes*

Problem 2 (35 points):

The discrete equivalent linear time invariant state model of a second order system is given by the following:

$$\begin{aligned} X(k+1) &= A X(k) + B U(k) \\ Y(k) &= C X(k) \end{aligned}$$

The matrices are:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \quad B^T = [0 \ 1] \quad ; \quad C = [2 \ 0]$$

- +10 a) What are the poles of the system in the z-plane?  $\frac{1}{2} \pm j0.86$
- +10 b) Is the given system fully observable? Justify. *yes R-beits*
- +15 c) Design a predictor estimator to have desired poles in the Z-plane located at 0. (Supply needed gain vector).  $L_p = [0.5 \ 0]$

*Difference Equation Model of estimator*

Problem 3 (35 points):

The discrete equivalent (T is 0.1 sec) linear time invariant state model of a continuous second order system is given by the following:

$$\begin{aligned} X(k+1) &= A X(k) + B U(k) \\ Y(k) &= C X(k) \end{aligned}$$

The matrices are:

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad B^T = [1 \ 1] \quad ; \quad C = [1 \ 2]$$

The desired given reference state vector is given by  $X_d(k)$  different from zero and corresponds to a given command input signal  $U_c(k)$ . Define the state error by  $e(k) = X(k) - X_d(k)$ .

- +5 a) Is the given system fully controllable? Justify. *yes @ exists*
- +15 b) Design a feedback gain matrix G such that  $U(k) = U_c(k) - G e(k)$  will force the closed loop system error to go to zero with desired poles in the Z-plane located at 0.4 and 0.6.  $G = [-0.25, 1.24]$
- +15 c) What are the obtained transient step response specifications of the closed loop system in the continuous domain?

*Handwritten calculations:*

$$s = \frac{1}{T} \ln(z) = \begin{bmatrix} -9.16 \\ -5.1 \end{bmatrix} \Rightarrow \zeta = 0.27$$

$$\left. \begin{matrix} \sigma > -1 \\ \omega > 0 \end{matrix} \right\} \Rightarrow \text{overdamped} \Rightarrow \zeta = 2 \cdot 0.167$$